

Borromean Objects, as exemplified by the group G_{168} of Klein's Quartic, linked with Moving Logic *

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The Klein's quartic (cf. [1],[2]) $X(7) = \{[x : y : z] \in P_2(\mathbb{C}) ; x^3y + y^3z + z^3x = 0\}$ is a smooth algebraic curve, and so is riemannian, and her genus is 3. Its group of homographic symetries is G_{168} , the maximal in genus 3. On $X(7)$ we can inscribe with a nice ternary symetry a borromean link without double points, and so the borromean is of genus 3. In fact we would like to go further and really understand $X(7)$ — or its group G_{168} — as a *borromean object* in a convenient category.

The notion of *borromean object* is defined precisely, and the case of G_{168} is explained by introducing three generators r , s and i playing symmetric roles (see [3] for a summary of the first properties of these r , s , i).

A borromean object in the category of sets is nothing else than a set B divided in three subsets, and so B is determined by its three components ; but this is not true in others contexts, for instance in the category of groups where components are not enough to determine a borromean object, the link between components provided by a borromean object being stronger than a simple concatenation.

We give other examples of borromean objects in rings (borromean rings !) or even in boolean algebras.

Finally we explain how these notions and constructions provide models for Moving Logic [3] — a kind of logic in which some 'points of view' operate as modalities — useful for discourse analysis [4]. Informally we can imagine that in $X(7)$ each of the three 'holes' is a point of view for enunciation, and the surface is the real world establishing symmetric communications between the holes ; and the same idea works for any abstract borromean object.

REFERENCES

- [1] F. Klein, *Über die Transformationen siebenter Ordnung der elliptischen Funktionen*, Math. Ann. 14 (1879), 428-471. Translate in [2].
- [2] S. Levy, *The Eightfold Way*, Cambridge U. Press, 1999.
- [3] R. Guitart, *Théorie cohomologique du sens*, SIC Amiens, 8 novembre 2003, compte-rendu 2004-10/Mars 2004, LAMFA CNRS UMR 6140, 39-47.
- [4] R. Guitart, *Moving Logic, from Boole to Galois*, Colloque International 'Charles Ehresmann : 100 ans', 7-9 octobre 2005, Cahiers Top. Go. Diff. Cat. XLVI-3, 196-198.

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